AESB 2320 Part 1 Exam March 8 2016

1. For flow in a tube, $f = \frac{1}{4}\frac{D}{L}\left(\frac{\Delta \mathcal{P}}{\frac{1}{2}\rho v^2}\right)$

Rocky thinks flow is laminar, i.e. that $f = \frac{16}{Re}$

For $Re = 3000$, in laminar flow, $f = \frac{16}{3000} = 0.00533$

In reality, for a hydraulically smooth tube, at $Re = 3000$,

$f \cong 0.011$ (BSLK Fig 6.2-2, BSL2 Fig 6.2-2)

f is about 2x larger (2.06x); i.e., ΔP is about 2x what it would be for this fluid in laminar flow. Therefore, Rocky thinks the fluid is 2x as viscous as it really is.

He thinks $\mu \cong 0.0206$

2. Initial note on problem 2. I had intended that students take the boundary condition at $r = R$ to be $\tau_{rz} = -\frac{F}{2\pi R L}$; I only mentioned the velocity V as the result of F. However, the problem statement was ambiguous, and all students took the B.C. as $v_z = V$ at $r = XR$. The problem can still be worked that way. First I give the solution with F as boundary condition, then V.

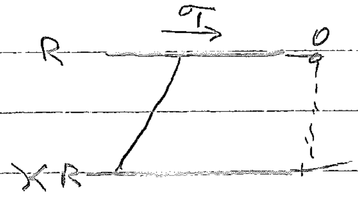
a) The geometry is cylindrical. There is no gravity or Δp. Thus the momentum balance of either flow in a tube (BSL Sect 2.3) or annulus (BSL Sect 2.4) applies w/ $\Delta \mathcal{P} = 0$.

In either case { tube: BSL 1 Eq. 2.3-10

annulus: BSL 1 Eq. 2.4-1

$$\frac{d}{dr}(r \tau_{rz}) = 0 \rightarrow r \tau_{rz} = C_1$$

B.C.: at $r = R$, $\tau_{rz} = -\frac{F}{2\pi R L}$ (minus, because momentum travels inward, in direction of decreasing r)*



Therefore $C_1 = RF/(2\pi R L) = F/(2\pi L)$

$$\tau_{rz} = \frac{-F}{2\pi L}\frac{1}{r}$$

τ_{rz} increases in magnitude as r decreases

b) Plug in Newton's law of viscosity

$$\tau_{rz} = -\frac{F}{2\pi L}\frac{1}{r} = -\mu\frac{dv_z}{dr}$$

$$\frac{dv_z}{dr} = \frac{F}{2\pi\mu L}\frac{1}{r}$$

integrate: $v_z = \frac{F}{2\pi\mu L}\ln r + C_1$

BC: $v_z = 0$ at $r = \kappa R$

$$0 = \frac{F}{2\pi\mu L}\ln(\kappa R) + C_1$$

$$C_1 = -\frac{F}{2\pi\mu L}\ln(\kappa R)$$

$$v_z = \frac{F}{2\pi\mu L}\ln(r/\kappa R)$$

c) rearranging, and noting that $v_z = V$ at $r = R$,

$$F = V(2\pi\mu L)/(\ln(R/\kappa R))$$

$$= \frac{V(2\pi\mu L)}{\ln\kappa}$$

[note similarity to conduction in cylindrical geometry]

Using $v_z = V$ at $r = R$ as B.C.

a) same as above, up to $\tau_{rz} = C_1/r$

no BC on τ, so proceed.

b) $\tau_{rz} = \frac{C_1}{r} = -\mu\frac{dv_z}{dr}$; $\frac{dv_z}{dr} = -\frac{C_1}{\mu r}$

$$v_z = -\frac{C_1}{\mu}\ln r + C_2$$

B.C. $v_z = 0$ at $r = \kappa R$: $0 = -\frac{C_1}{\mu}\ln(\kappa R) + C_2$

$$C_2 = \frac{C_1}{\mu}\ln(\kappa R)$$

$$\rightarrow v_z = \frac{C_1}{\mu}\left(\ln(\kappa R/r)\right)$$

$v_z = V$ at $r = R$

$$V = \frac{C_1}{\mu}\ln\left(\frac{\kappa R}{R}\right) = \frac{C_1}{\mu}\ln(\kappa)$$

$$C_1 = V\mu/(\ln\kappa)$$

$$v_z = \left(\frac{V\mu}{\ln\kappa}\right)\frac{1}{\mu}\ln\left(\frac{\kappa R}{r}\right) = \frac{V}{\ln\kappa}\ln\left(\frac{\kappa R}{r}\right)$$

[to finish (a), $\tau_{rz} = \left(\frac{V\mu}{\ln\kappa}\right)\left(\frac{1}{r}\right)$ *

c) Since $\tau_{rz} = \left(\frac{V\mu}{\ln\kappa}\right)\frac{1}{r}$, $\tau_{rz} = \left(\frac{V\mu}{\ln\kappa}\right)\frac{1}{R}$ at $r = R$

Force $= \tau_{rz} * \text{area} = \tau_{rz}(2\pi RL) = \left(\frac{V\mu}{\ln\kappa}\right)\frac{1}{R}(2\pi RL) = \frac{V\mu}{\ln\kappa}2\pi L$

* [note: since $(\ln\kappa) < 0$, $\tau_{rz} < 0$]

3. $K = (D_p^2/150)\{\varepsilon^3/(1-\varepsilon)^2\} = \frac{(0.0005)^2}{150}\frac{(0.35)^3}{(0.65)^2} = 1.69 \cdot 10^{-10}\ m^2$ (about 170 darcy)

4. One could put surface "1" at either the top of the liquid in the tank or just above the outlet. One gets the same answer. I put surface "1" at the top of the liquid in the tank. BSLK Eq. 7.5-11 (BSL2 Eq. 7.5-10) gives

$$\tfrac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1) + \frac{P_2 - P_1}{\rho} = \hat{W}_m - \sum_i\left(\tfrac{1}{2}v^2\frac{L_4}{D_h}f\right) - \sum\left(\tfrac{1}{2}v^2 e_v\right)$$

- Call velocity in the wider pipe V_w; in the narrower tube, V_n. Note $V_2 = V_w$.

$$V_w = Q/[\pi(0.025)^2] = 0.003/(\pi(0.025)^2) = 1.53\ m/s = V_2$$
$$V_n = [Q/\pi(0.01)^2] = 0.003/(\pi \cdot 10^{-4}) = 9.55\ m/s$$

- first term : $\tfrac{1}{2}(v_2^2 - v_1^2) = \tfrac{1}{2}\{(1.53)^2 - 0\} = 1.17$
- 2nd " : $g(h_2 - h_1) = -9.82(3+1) = -39.28$
- 3rd " : $\frac{P_2 - P_1}{\rho} = \frac{10^5 - P_0}{1100}$
- 4th " : $\hat{W}_m = 0$
- 5th " : two sections of pipe.

narrow pipe: $Re = \frac{(0.02)9.55(1100)}{0.003} = 7.00 \cdot 10^4$

$K/D = 8 \cdot 10^{-5}/(0.02) = 0.004$

from Fig 6.2-2, $f \cong 0.0078$

$\tfrac{1}{2}v^2\frac{4L}{D_h}f = \tfrac{1}{2}(9.55)^2 \frac{4 \cdot (1+3)}{(0.02)}(0.0078) = 285$

wide pipe: $Re = \frac{(0.05)(1.53)\,1100}{0.003} = 2.8 \cdot 10^4$

$K/D = 8 \cdot 10^{-5}/(0.05) = 0.0016$

from Fig 6.2-2, $f \cong 0.0069$

$\tfrac{1}{2}v^2\frac{4L}{D_h}f = \tfrac{1}{2}(1.53)^2\frac{4 \cdot 5}{(0.05)}(0.0069) = 3.23$

- 6th term: sudden contraction at tank: $\tfrac{1}{2}v_n^2 e_v = \tfrac{1}{2}(9.55)^2(0.45) = 20.5$

90° elbow: $\tfrac{1}{2}v_n^2 e_v = \tfrac{1}{2}(9.55)^2(1.6) = 72.9$

expansion to wider pipe: $\left[\frac{1}{((0.02)^2/(0.05)^2)} - 1\right]^2 = e_v = (6.25-1)^2 = 27.6$

$\tfrac{1}{2}v_w^2 e_v = \tfrac{1}{2}(1.53)^2(27.6) = 32.3$

All together: $1.17 - 39.28 + \left(\frac{10^5 - P_0}{1100}\right) = -285 - 3.23 - 20.5 - 72.9 - 32.3$

$$(10^5 - P_0) = -375.8\ (1100)$$

$$P_0 = 5.13 \cdot 10^5 \quad (\sim 4.84 \text{ bar pressure})$$